Modified Berenger PML Absorbing Boundary Condition for FD-TD Meshes

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***Abstract*—A new modified, perfectly matched layer absorbing boundary condition (MPML ABC) is presented. In the MPML, the introducing of extra degrees of freedom provides the possibility of adjusting the parameters of nonphysical material absorber (PML) for the purpose of enhancing the attenuation rate of the evanescent modes. Compared to Berenger PML, the MPML is more efficient in absorbing the evanescent energy and keeps the same performance for propagation modes. The sophisticated properties of MPML ABC result in the reduction of the thickness of the matched layers and their closer allocation, leading to better accuracy simulations and less computer burdens in FD-TD modeling.**

I. Introduction

The effective absorbing boundary condition (ABC) is one of the most primary interests in finite-difference time-domain (FD-TD) solutions of Maxwell's equations to model open-region electromagnetic problems. The revolutionary Berenger PML ABC gives the reflection coefficients as low as 1/3000th those of standard second- and third-order analytical ABC's such as Mur [1], [2]. More recently, evidence is presented that the PML ABC is effective even for the evanescent energy present below cutoff in perfectly conducting waveguides and the multimode propagation present in dielectric waveguides [3].

However, although PML is matched to evanescent modes, it is impossible to enhance the attenuation rate of the evanescent modes through the adjusting parameters of the matched layers. The reason is that the attenuation rate of the evanescent modes in matched layers is the same as that in free space. For the evanescent modes with low attenuation rate, in order to get small reflection, sufficient thickness of the matched layers is required, for example, 16 layers in [3]. Alternatively, the matched layers should be located far from the sources and scattering objects. In either case, the computer storage and computer time will increase significantly.

To overcome this trouble, in this letter we propose a new modified PML (MPML). In the MPML, the introduction of extra degrees of freedom provides the possibility of adjusting the parameters of the matched layers for the purpose of enhancing the attenuation rate of the evanescent modes in the matched layers. Compared to Berenger PML, the MPML is more efficient in absorbing the evanescent energy and keeps the same performance for propagation modes. With MPML, it is possible to reduce the thickness of the matched layers and to allocate them close to the sources or scattering objects even down to two space steps.

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II. Theory

The key point of Berenger PML ABC is the creation of nonphysical absorber adjacent to the outer grid boundary by splitting the field component and introducing a new degree of freedom [1], [2]. The difference between PML and MPML is the extra degrees of freedom related to ϵ_r and μ_r are introduced.

For clear explanation, the 2-D TE case is considered and the same notations in [1] and [2] are used. Equations (2a)–(3b) in [2] are modified as

$$\epsilon_0\epsilon_y\frac{\partial E_x}{\partial t}+\sigma_y E_x=\frac{\partial}{\partial y}(H_{zx}+H_{zy}) \tag{1a}$$

$$\epsilon_0\epsilon_x\frac{\partial E_y}{\partial t}+\sigma_x E_y=-\frac{\partial}{\partial x}(H_{zx}+H_{zy}) \tag{1b}$$

$$\mu_0\mu_x\frac{\partial H_{zx}}{\partial t}+\sigma_x^* H_{zx}=-\frac{\partial E_y}{\partial x} \tag{2a}$$

$$\mu_0\mu_y\frac{\partial H_{zy}}{\partial t}+\sigma_y^* H_{zy}=\frac{\partial E_x}{\partial y} \tag{2b}$$

$$H_z=H_{zx}+H_{zy}. \tag{3}$$

Note that when $\epsilon_x=\epsilon_y=\mu_x=\mu_y=1$, the above equations (1a)–(2b) degenerate to (2a)–(3b) in [2]. It can be shown that

$$\psi=\psi_0 e^{jw\left(t-\frac{\epsilon_x x\cos\Phi+\epsilon_y y\sin\Phi}{CG}\right)}e^{-\frac{\sigma_x\cos\Phi}{\epsilon_0 CG}x}e^{-\frac{\sigma_y\sin\Phi}{\epsilon_0 CG}y} \tag{4a}$$

$$Z=G\sqrt{\mu_0/\epsilon_0} \tag{4b}$$

where Z is the wave impedance, C is the speed of light, Φ is the angle between the wave electric field vector and the y axis, and

$$G=\sqrt{\omega_x\cos^2\Phi+\omega_y\sin^2\Phi} \tag{5a}$$

$$\omega_x=\frac{\epsilon_x-j\sigma_x/\omega\epsilon_0}{\mu_x-j\sigma_x^*/\omega\epsilon_0},\quad \omega_y=\frac{\epsilon_y-j\sigma_y/\omega\epsilon_0}{\mu_y-j\sigma_y^*/\omega\epsilon_0}. \tag{5b}$$

Now, let the following expressions be satisfied:

$$\frac{\sigma_x}{\epsilon_0} = \frac{\sigma_x^*}{\mu_0}, \; \frac{\sigma_y}{\epsilon_0} = \frac{\sigma_y^*}{\mu_0} \tag{6a}$$

$$\mu_x = \epsilon_x, \; \mu_y = \epsilon_y. \tag{6b}$$

Then, ω_x, ω_y and G equal one at any frequency, and the wave components and the wave impedance of (4) become

$$\psi = \psi_0 e^{jw\left(t - \frac{\epsilon_x x \cos \Phi + \epsilon_y y \sin \Phi}{C}\right)} e^{-\frac{\sigma_x \cos \Phi}{\epsilon_0 C} x} e^{-\frac{\sigma_y \sin \Phi}{\epsilon_0 C} y} \tag{7a}$$

$$Z = \sqrt{\frac{\mu_0}{\epsilon_0}}. \tag{7b}$$

In a 2-D TE grid (x and y coordinates), a normal free-space FD-TD computational zone is surrounded by a MPML backed by perfectly conducting walls. At both the left and right sides of the grid ($x_{\min}$ and $x_{\max}$), each MPML has σ_x, σ_x^*, ϵ_x and μ_x matched according to (6) along with $\sigma_y = \sigma_y^* = 0$ and $\epsilon_y = \mu_y = 1$ to permit reflectionless transmission across the vacuum-MPML interface. This can be easily shown by generalizing the Snell's law related to PML media. At both the lower and upper sides of the grid ($y_{\min}$ and $y_{\max}$), each MPML has σ_y, σ_y^*, ϵ_y and μ_y matched according to (6) along with $\sigma_x = \sigma_x^* = 0$, $\epsilon_x = \mu_x = 1$. At the four corners of the grid where there is overlap of two MPML's, all parameters of MPML are present (σ_x, σ_x^*, σ_y, σ_y^*, ϵ_x, μ_x, ϵ_y, and μ_y) and set equal to those of the adjacent MPML's.

If σ is chosen to be $\sigma = \sigma_{\max}(\rho/\delta)^n$, where δ is the PML thickness and σ is either σ_x or σ_y. For the propagating modes, then we may have the MPML reflection factor

$$R(\theta) = e^{-2\sigma_{\max}\delta \cos\theta/(n+1)\epsilon_0 C} \tag{8}$$

and the theoretical reflection coefficient at normal incidence for the MPML over PEC is

$$R(0) = e^{-2\sigma_{\max}\delta/(n+1)\epsilon_0 C}. \tag{9}$$

From (7a) and (8), it can be seen that no matter PML or MPML, both the decay and the reflection coefficient are the same.

For the evanescent modes in y direction, Φ takes the form of

$$\Phi = -j\alpha. \tag{10}$$

α is real number and is larger than zero.

Equation (7a) may be rewritten as

$$\psi = \psi_0 e^{jw\left(t - \frac{\mathrm{ch}\alpha}{C} x\right)} e^{j\frac{\sigma_y \mathrm{sh}\alpha}{\epsilon_0 C} y} e^{-\epsilon_y k \mathrm{sh}\alpha y} \tag{11}$$

where $k = \omega/C$. The evanescent mode reflection coefficient R_{em} is

$$R_{em} = e^{-2\epsilon_y k \mathrm{sh}\alpha y}. \tag{12}$$

To avoid the sudden change at the interface between vacuum and MPML, ϵ_y is chosen to be $\epsilon_y = 1 + \epsilon_{\max}(\rho/\delta)^n$. It can be seen that R_{em} may be reduced by increasing ϵ_y or $\epsilon_{\max}$. This fact confirms that the MPML is more efficient for evanescent modes. Numerical experiments given in the following section support the theory. Finally, it should be mentioned that too large value of $\epsilon_{\max}$ may result in strong numerical dispersion, according to our experiences, $\epsilon_{\max} < 10$ is suggested.

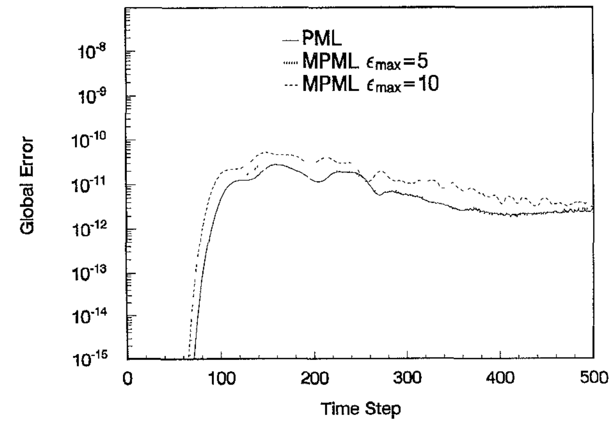


Fig. 1. Global error energy (square of the magnetic field error at each grid cell summed throughout the entire grid) within the 100 × 50 cell 2-D TE FD-TD grid for both the 16-cell-thick PML ABC and MPML ABC, plotted as a function of time step number on a logarithmic vertical scale.

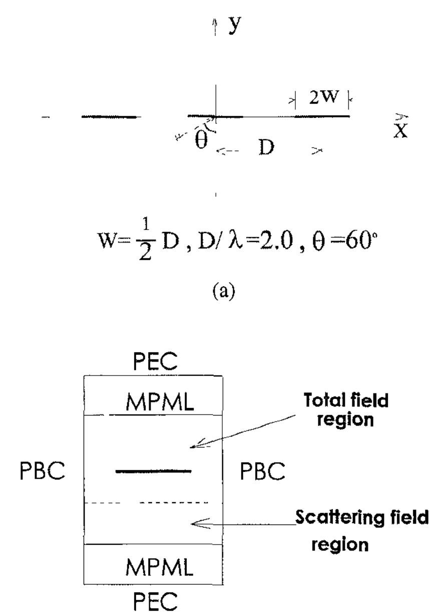
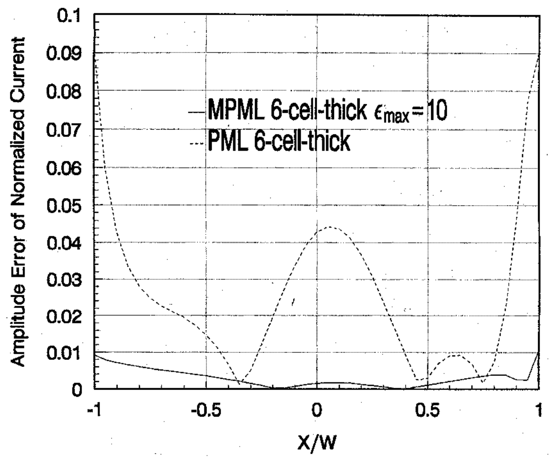


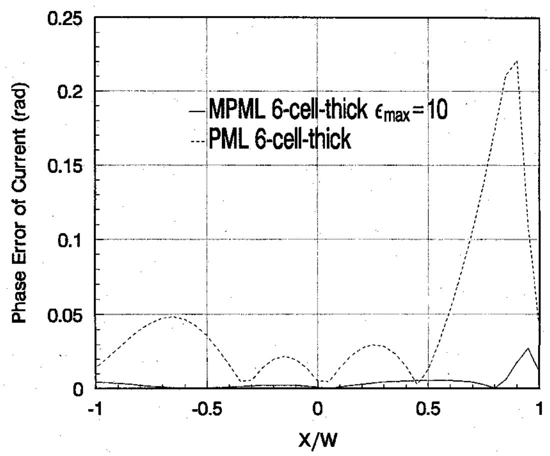
Fig. 2. (a) Geometry of two-dimensional frequency selective surface. (b) Computational domain and boundary conditions.

III. Numerical Experiments

To verify the effectiveness of the MPML to the propagating modes first, we conducted numerical experiments that implemented the PML ABC and MPML ABC to the same case given in Fig. 1 of [2] under the same excitation as described in [2]. The global error in MPML is slightly larger than that in PML. The reason is that for this case, the space-stepping Δ is only about $\lambda/6$. Consider that in practical cases ($\Delta \approx \lambda/20$ in this case), both MPML and PML possess almost the same performance as shown in Fig. 1. To compare the performance of PML ABC and MPML ABC to the evanescent modes, they are implemented to the scattering problem of 2-D, as shown in Fig. 2. The excitation is monochromatic TM wave. In this case, the evanescent modes are known, thus it is easier to



(a)



(b)

Fig. 3. Error of current density distribution on the strip of FSS for both the 6-cell-thick, $\epsilon_{\max} = 10$ MPML and 6-cell-thick PML. (a) Amplitude. (b) Phase.

observe the absorbing performance of these modes. In the figure, two side walls satisfy the Floquet periodic boundary condition (PBC), the implementation of PBC is given in [4]. The top and bottom walls outside the MPML's are perfectly conducting. According to the analysis and numerical testing, 16-cell-thick PML away from the strip of FSS by 12-space stepping may precisely simulate the free space. Therefore, numerical results from this situation may serve as reference values. Fig. 3 shows the amplitude and phase error of current density distribution on the strip of FSS for both the 6-cell-thick, $\epsilon_{\max} = 10$ MPML and 6-cell-thick PML. In the figure, intervals between strip and top PML and between top and bottom PML are 2Δ and 4Δ, respectively.

It is seen that the MPML may reduce the amplitude or phase error by one order. The energy conservation check shows that the error for PML is 4%; for MPML is only 0.1%. Our other results also show that when the PML cell thick goes down, the improvement will be more significant.

IV. Conclusion

We have demonstrated the use of MPML ABC for highly absorbing the evanesent energy in FD-TD grids. The sophisticated properties of MPML ABC result in the reduction of the thickness of the matched layers and the closer allocation of them, leading to better accuracy simulations and less computer burdens in FD-TD modeling. The extension of MPML to 3-D is straightforward, as is done in [3].

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